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ESTIMATION OF PETROLEUM RESERVOIR PROPERTIES

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Abstract

We present an algorithm for estimating the absolute and relative permeabilities in petroleum reservoir models based on regularization and spline approximation. A computational example is included.

1. Introduction

Once wells have been drilled down into a reservoir containing recoverable petroleum, the local properties of the reservoir rocks and fluids must be determined. A variety of complex acoustical, electronic, and magnetic techniques are available that can be used to determine the local properties of the formation and fluids in the neighborhood of the well. Estimates of the reservoir properties are needed, however, throughout the entire reservoir, not just at the wells, in order to simulate various production strategies to try to optimize the recovery of the petroleum. To estimate the properties of the reservoir, past production histories are simulated. The properties are determined as those that produce the closest possible match of the observed and predicted histories. This so-called history-matching process has been addressed in the petroleum, hydrology, and mathematics literature for some 20 years or so.

In the early stages of production of a petroleum reservoir, it often can be assumed that the reservoir contains only a single fluid, oil. In that case the reservoir behavior is described by a single linear parabolic PDE for pressure. The reservoir parameters that enter the equation, and are subject to estimation, are the rock porosity ϕ and the absolute permeability k, both of which vary with location in the reservoir. Generally, one must account for the fact that oil and water are present together in petroleum reservoirs, and the resulting reservoir model consists of two coupled nonlinear PDEs. In addition to the porosity ϕ and absolute permeability k, the two-phase case is characterized by the *relative permeabilities* k_{ro} and k_{rw} (o referring to oil, w referring to water) that are presumed to be functions of the local fluid saturation in the medium. The precise values of the two relative permeabilities usually are not known.

The essential difficulties in the petroleum reservoir inverse problem are twofold. First, the reservoir properties are spatially varying, and the estimation of a spatially varying permeability is well known to be an ill-posed problem [1–4]. Second, the oil-water reservoir is a highly nonlinear system, and rigorous results concerning its inverse problems do not exist.

The ill-posed nature of the single-phase permeability estimation problem has been attacked by Bayesian approaches [5,6], regularization [3,4,7–10], and spline approximation [11]. While the Bayesian approach requires *a priori* statistical information on the unknown parameters that may not be generally available and while spline approximation in and of itself does not guarantee the problem to be well-posed, the regularization approach offers both rigorous stability and convenient computational implementation. The first step of the regularization formulation is to measure the non-smoothness of the parameter by its norm in an appropriate Hilbert space, called the stabilizing functional, and then to seek the value of the parameter that minimizes the weighted sum of the least-squares discrepancy term and the stabilizing functional. In previous applications of regularization to the petroleum reservoir inverse problem, Lee *et al.* [9] estimated absolute permeability and porosity in a single-phase reservoir and Lee and Seinfeld [12] estimated the absolute permeability in a two-phase reservoir.

The object of the present paper is to develop an algorithm for the simultaneous estimation of absolute and relative permeabilities in two-phase petroleum reservoirs.

2. Mathematical Model of Two-Phase Petroleum Reservoir

Consider a two-dimensional oil-water reservoir that has sufficiently large areal extent so that we can assume that the pressure change and hence the flow in the vertical direction are negligible compared to those in the other two directions [13]. Assuming that the oil and water phases are immiscible, the equations of mass conservation for the oil and water phases are

$$\frac{\partial}{\partial t}(\rho_o \phi S_o) + \nabla \cdot (\rho_o \mathbf{v}_o) = \sum_{\kappa=1}^{N_w} \rho_o q_{o\kappa} \frac{\delta(x-x_\kappa)\delta(y-y_\kappa)}{h} \quad (1)$$

$$\frac{\partial}{\partial t}(\rho_w \phi S_w) + \nabla \cdot (\rho_w \mathbf{v}_w) = \sum_{\kappa=1}^{N_w} \rho_w q_{w\kappa} \frac{\delta(x-x_\kappa)\delta(y-y_\kappa)}{h} \quad (2)$$

for $(x,y) \in \Omega$ and $0 < t < T$, where S_o and S_w, the volume fractions of oil and water with respect to the total fluid volume, called oil and water saturations, respectively, satisfy $S_o \equiv 1 - S_w$. The oil-water reservoirs that do not include gas phase generally are slightly compressible systems; i.e., the porosity, ϕ, and the density of oil, ρ_o, and water, ρ_w, are weak functions of pressure. It is customary that the functional dependencies are given by $c_f = (1/\phi)(d\phi/dp)$, $c_o = (1/\rho_o)(d\rho_o/dp)$, and $c_w = (1/\rho_w)(d\rho_w/dp)$ where c_f, c_o, and c_w denote the compressibilities of rock, oil, and water and are assumed to be constant over the entire region of pressure change of the reservoir. The volumetric flow rates of the water and oil phases at the wells located at (x_κ, y_κ) are denoted by $q_{o\kappa}$ and $q_{w\kappa}$, $\kappa = 1, \ldots, N_w$. For injection wells, $q_o = 0$ and $q_w > 0$. For production wells, q_o and q_w are negative, and the ratio q_w/q_o is proportional to the ratio of local flow velocities of water to oil at the bottom of wells. The thickness of the reservoir, h, is assumed to be constant over the whole reservoir domain. The linear velocities of the oil and water phases are assumed to be described by Darcy's Law,

$$\mathbf{v}_o = -\frac{kk_{ro}}{\mu_o}\nabla p \qquad \mathbf{v}_w = -\frac{kk_{rw}}{\mu_w}\nabla p\,, \quad (3)$$

where the absolute permeability k is a parameter characterizing the fluid conductivity of a porous medium, μ_o and μ_w are the viscosities of oil and water, respectively, and the relative permeabilities of oil and water, k_{ro} and k_{rw}, respectively, are assumed to be functions of fluid (water) saturation within the porous medium independent of flow rate and fluid properties. Widely used functional forms of the relative permeabilities, and those employed in this study, are

$$k_{ro}(S_w) = a_o \left(\frac{1 - S_{ro} - S_w}{1 - S_{ro} - S_{iw}} \right)^{b_o} \quad (4)$$

$$k_{rw}(S_w) = a_w \left(\frac{S_w - S_{iw}}{1 - S_{ro} - S_{iw}} \right)^{b_w} \tag{5}$$

for $S_{iw} \le S_w \le 1 - S_{ro}$ where irreducible (or connate) water saturation, S_{iw}, and residual oil saturation, S_{ro}, are the lower bounds of S_w and S_o, respectively, under which water and oil become immobile with reasonable pressure gradients. The relative permeabilities are each less than unity, and typically, their sum is also less than unity for $S_{iw} < S_w < 1 - S_{ro}$. Eqs. (1–5) together with the no-flux boundary condition,

$$\mathbf{n} \cdot \nabla p = 0 \ , \tag{6}$$

for $(x, y) \in \partial\Omega$ and $0 < t < T$, and the given initial conditions

$$p(x, y, 0) = p_0(x, y) \tag{7}$$

$$S_w(x, y, 0) = S_{w0}(x, y) \tag{8}$$

for $(x, y) \in \Omega$ describe the water-driven oil recovery process for a petroleum reservoir with an impermeable boundary. Eqs. (1–8) are solved numerically using finite difference approximation. Physically, these equations describe the movement of both phases, usually as water is intentionally pumped down certain wells to drive the oil in place toward other wells where it is produced. When the water breaks through at the production wells, the displacement process is considered to be complete.

3. The Inverse Problem

It is desired to estimate simultaneously the absolute permeability, k, and the relative permeabilities, k_{ro} and k_{rw}, from data normally available at wells that have been drilled into the reservoir. Since k_{ro} and k_{rw} are assumed to be given by Eqs. (4) and (5), their estimation reduces to that of the unknown constant parameters a_o, a_w, b_o, and b_w. In general, a_o and a_w can be determined if the values of k_{ro} and k_{rw} are known at two points such as at the connate water or residual oil saturations. Thus, b_o and b_w are the more uncertain and will be the subject of estimation here. The measured data consist of the pressure at N_o wells and at N_t discrete times over $0 < t < T$ and of the water fraction of the total flow at each well,

$$f_w = \frac{k_{rw}/\mu_w}{k_{rw}/\mu_w + k_{ro}/\mu_o} \ . \tag{9}$$

The usual least-squares objective function consists of two contributions, one each from the pressure and the water flow observations. We define σ_p^2 as the mean-square error between the calculated and measured pressure data

$$\sigma_p^2 = \frac{1}{N_o N_t} \sum_{n=1}^{N_t} \sum_{\nu=1}^{N_o} \left(p(x_\nu, y_\nu, t_n) - p^{\mathrm{obs}\,n}_{\ \ \nu} \right)^2 \tag{10}$$

where $(x_\nu, y_\nu) \in \Omega$, $\nu = 1, \ldots, N_o$ denote the locations of the observations, that is, the wells, and t_n, $n = 1, \ldots, N_t$ are the observation times. Similarly, we define σ_f^2 as the mean-square error in the water flow data,

$$\sigma_f^2 = \frac{1}{N_o N_t} \sum_{n=1}^{N_t} \sum_{\nu=1}^{N_o} \left(f_w(x_\nu, y_\nu, t_n) - f_w^{\mathrm{obs}\,n}{}_{\nu} \right)^2 \ . \tag{11}$$

Then the least-squares objective function is given by a weighted sum of the two contributions

$$J_{LS}(k, b_o, b_w) = W_p \sigma_p^2 + W_f \sigma_f^2 \tag{12}$$

where W_p and W_f are the weighting coefficients for the pressure and flow-rate terms, respectively.

The conventional least-squares identification problem is to estimate $k(x, y)$, b_o, and b_w to minimize J_{LS}. The spatial variation of k leads to an ill-posed inverse problem, and hence we turn to a regularization formulation. Kravaris and Seinfeld [4,8] introduced the concept of regularization for the estimation of coefficients in PDEs. Regularization of a problem refers to solving a problem related to the original problem, called the regularized problem, the solution of which both is more "regular" and approximates the solution of the original problem. In Tikhonov's regularization formulation [14], the measure of non-smoothness of the parameter being estimated, called the stabilizing functional, is represented by a norm of the parameter in an appropriate Hilbert space, for example,

$$J_{ST}(k) = ||k||^2_{H^3(\Omega)} \ , \tag{13}$$

where the Sobolev space $H^3(\Omega)$ is the set of functions that are square-integrable over Ω and have square-integrable derivatives up to order 3. More precisely, Tikhonov's stabilizing functional is given by

$$J_{ST}(k) = \sum_{m=0}^{3} \varsigma_m \iint_\Omega \sum_{\nu=0}^{m} \binom{m}{\nu} \left(\frac{\partial^m k(\xi, \eta)}{\partial \xi^\nu \partial \eta^{m-\nu}} \right)^2 d\xi d\eta \tag{14}$$

where convenient dimensionless variables are $\xi = N_x x / x_L$ and $\eta = N_y y / y_L$, where x_L and y_L are the lateral reservoir dimensions and N_x and N_y are the number of PDE grid cells employed along x- and y-directions, respectively. The conditions for the coefficients ς_m are $\varsigma_0 > 0$, $\varsigma_1 > 0$, $\varsigma_2 > 0$, and $\varsigma_3 > 0$ [15]; or $\varsigma_0 \ge 0$, $\varsigma_1 \ge 0$, $\varsigma_2 \ge 0$, and $\varsigma_3 > 0$ [14]. As Trummer [16] has pointed out, using the stabilizing functional that includes the Euclidean norm of the parameter itself leads to the underestimation of the parameter. Locker and Prenter [17] suggested the use of a stabilizing functional with a differential operator. Lee and Seinfeld [12] used the stabilizing functional with the gradient operator (∇) so that it does not include the Euclidean norm term ($\varsigma_0 \equiv 0$ in Eq. (14)) for the estimation of absolute permeability.

The regularization formulation of the inverse problem seeks the minimum of the smoothing functional,

$$J_{SM}(k, b_o, b_w; \beta) = J_{LS}(k, b_o, b_w) + \beta J_{ST}(k) \ , \tag{15}$$

where β is the regularization parameter that represents the relative importance given to J_{ST}. In the present problem, J_{LS} is composed of the two terms as shown in Eq. (12); hence, J_{SM} includes three quantities, $W_p \sigma_p^2$, $W_f \sigma_f^2$, and βJ_{ST}, where two of the three weighting coefficients W_p, W_f, and β must be determined independently. W_f / W_p can be chosen as the ratio $\bar{\sigma}_p^2 / \bar{\sigma}_f^2$, where $\bar{\sigma}_p^2$ and $\bar{\sigma}_f^2$ denote the variances associated with the pressure and production data measurements, respectively [18]. In the present study, $\bar{\sigma}_p^2 / \bar{\sigma}_f^2$ is assumed to be known and W_f / W_p is chosen as that value. An important question regarding the regularization method is determining a suitable value of β for the given noisy data, especially where the noise level may or may not be known. The value of β is chosen in several different ways [14,19,20]. Miller suggests that β be determined from the ratio of an upper bound of the measurement error to an upper bound of the measure of non-smoothness. Craven and Wahba [20] used the method of generalized cross validation (GCV) to determine the regularization parameter. Since GCV requires parametric sensitivity information, this method is not practical for such a large-scale problem like reservoir parameter estimation. Lee and Seinfeld [12] developed an algorithm based on Miller's idea that determines the regularization parameter automatically during the estimation process without requiring *a priori* information.

The absolute permeability in a two-phase reservoir is primarily estimated from the pressure data [21,22]. Thus, we can determine β/W_p from the ratio of an upper bound of σ_p^2 to an upper bound of J_{ST}. In practice, these values usually are not known, and Lee and Seinfeld [12] used the values of J_{ST} and the pressure discrepancy of the results of the non-regularized ($\beta = 0$) estimation to determine β. Without loss of generality W_p will be specified as $1/\bar{\sigma}_p^2$.

Spline approximation of spatially varying parameters has several merits including a built-in smoothing and computational convenience [8,11]. The spline approximation of the spatially varying absolute permeability is given by

$$k(x,y) = \sum_{l_y=1}^{N_{ys}} \sum_{l_x=1}^{N_{xs}} \chi^{*4}\left(4 - l_x + \frac{x}{\Delta x_s}\right) \chi^{*4}\left(4 - l_y + \frac{y}{\Delta y_s}\right) W_l \tag{16}$$

where $\chi^{*4}(\theta)$ is cubic B-spline function, Δx_s and Δy_s are the grid spacings for the spline approximation and $l = l_x + N_{xs}(l_y - 1)$ for $l_x = 1, \ldots, N_{xs}$ and $l_y = 1, \ldots, N_{ys}$.

The theory of regularization does not suggest any guidelines about the highest order of the derivative term that is included in Eq. (14). It is clear that in the case of discrete regularization with spline approximation, the choice of Sobolev space is closely related to the choice of spline function. We choose the Sobolev space $H^3(\Omega)$ so that all nontrivial derivatives of cubic B-spline functions contribute to the evaluation of the stabilizing functional.

The problem is to estimate the spline coefficients, W_l, $l = 1, \ldots, N_s$, and the dimensionless exponents, b_o and b_w, in the relative permeability expressions, that minimize the smoothing functional J_{SM}.

To estimate (k, b_o, b_w) simultaneously, the following 3-step algorithm will be used assuming that no *a priori* information is available for the spatial variation of $k(x,y)$ and β.

Step 1 Assuming that $k(x,y) = \bar{k}$ over the whole domain, find $(\bar{k}, b_o, b_w)$ that minimize J_{LS}.

Step 2 Starting from $W_l = \bar{k}$, $l = 1, \ldots, N_s$, calculated from step 1, minimize J_{LS} with respect to $(\mathbf{W}, b_o, b_W)$. Compute $\beta = W_p\sigma_p^2/J_{ST}$ at convergence.

Step 3 Using β and starting from $(\mathbf{W}, b_o, b_w)$ determined in step 2, minimize J_{SM} with respect to $\mathbf{W}$, b_o, and b_w.

Step 2 of the algorithm is the conventional least-squares estimation of k by spline approximation, and of b_o and b_w, that gives the best fit of observed pressure and flow data. The major contribution of step 3 in the algorithm is to alleviate the ill-conditioning of the estimated k by a regularization. Generally, the exponents of the relative permeabilities, b_o and b_w, will not change significantly in step 3. In practice, therefore, step 3 can usually be replaced by

Step 3′

Using β, b_o, and b_w and starting from $\mathbf{W}$ determined in step 2, minimize J_{SM} with respect to $\mathbf{W}$.

In step 3′ the smoothing functional J_{SM} is minimized with respect to the single set of parameters, $\mathbf{W}$, and the minimization can be carried out by a general multivariate gradient algorithm. The partial conjugate gradient method of Nazareth [23] is chosen, as it is suitable for a large-scale minimization.

For the numerical implementation of the stabilizing functional with the gradient operator, J_{ST} with $\varsigma_0 = 0$ in Eq. (14), the weighting coefficients ς_m, $m = 1$, 2, and 3, need to be specified. Since the integration in Eq. (14) is based on the length scales of discretization of the PDEs, x_L/N_x and y_L/N_y, the grid spacings for the reservoir PDE, ς_ms of the derivative terms can be chosen as $\varsigma_1 = \varsigma_2 = \varsigma_3 = 1$.

4. Computational Example

In order to test the performance of the algorithm thoroughly, we will introduce a hypothetical reservoir for which the true properties are assumed to be known. The assumed fluid and reservoir properties are shown in Table I. The assumed true absolute permeability distribution is given by

$$k(x,y) = 0.3 - 0.1 \sin\left(\frac{2\pi x}{x_L}\right) \sin\left(\frac{\pi y}{y_L}\right) \tag{17}$$

in units of darcies (1 darcy $= 0.987 \times 10^{-12}$ m^2) for $(x,y) \in \Omega$. The location of wells and the true absolute permeability contour map are shown in Figure 1. The governing PDEs (1–9) are solved on a 15×10 mesh with the time stepsize of 23.1 days. The absolute permeability k is spline approximated on a 15×10 mesh. The observation data are taken from 9 observation wells that include 2 production wells with observation time interval 23.1 days and perturbed by uniformly distributed random numbers, with zero mean and standard deviations 0.34 atm and 0.0085 for p and f_w, respectively. These noisy data are then used to attempt to recover (k, b_o, b_w).

Over a period of 9.5 years, 150 pressure and 150 production data are taken at each of the 9 observation wells and $(\mathbf{W}, b_o, b_w)$ is estimated using the suggested 3-step algorithm. The results of the estimation are summarized in Table II. The first step is to estimate the set $(\bar{k}, b_o, b_w)$ that minimizes J_{LS}, where $\bar{k}$ denotes a spatially uniform k. Although the resultant $\bar{k}$ is not an acceptable estimate of a spatially varying k in most cases, it is a reasonable average of the spatially varying k. Two different sets of $(\bar{k}, b_o, b_w)$, (0.2 darcies, 1.5, 1.5) and (0.4 darcies, 3.0, 3.0) were chosen as the starting point of this step. The convergent results, (0.289 darcies, 2.09, 2.51) and (0.286 darcies, 2.06, 2.48), show good agreement, indicating the robustness of this step. In Figure 2, $\bar{k}k_{rw}(S_w)$, $\bar{k}k_{ro}(S_w)$, and $f_w(S_w)$ calculated from these values are depicted by the solid lines. This step makes the remainder of the algorithm insensitive to the choice of the initial guess $(\bar{k}, b_o, b_w)$. The next step is the pure least-squares estimation of (k, b_o, b_w) with $\beta = 0$, where k is represented by the set of spline coefficients $\mathbf{W}$. In this step, σ_p and σ_f decrease substantially and approach those calculated from the true (k, b_o, b_w). The estimated k is shown in Figure 3 and $(b_o, b_w) = (1.98, 2.50)$. From the resultant $W_p\sigma_p^2$ and J_{ST}, $\beta = 2.63$ darcies^{-2}. Step 3 is the final regularized estimation of (k, b_o, b_w) with β determined from step 2. The resultant k is shown in Figure 3 and $(b_o, b_w) = (1.98, 2.50)$. Comparison of the k contours in Figure 3 shows the smoothing effect of regularization on the "hump" near the lower right corner of the reservoir. As an alternative of step 3, step 3′ is the regularized estimation of $\mathbf{W}$, while b_o and b_w are fixed to the values determined by step 2 and the same β is used as step 3. The contours of the resultant k are shown in Figure 3, which shows more smoothing effect compared to that of step 3. Both the discrepancy and the stabilizing functional terms are smaller than those of step 3, while step 3′ required more computing time. Throughout the estimation process, (b_o, b_w) is estimated accurately even in step 1. The entire algorithms, steps 1, 2, and 3, required 63 and 74 iterations (solutions of state and adjoint PDEs), corresponding to 252 and 297 seconds of computing time; and steps 1, 2, and 3′, 66 and 77 iterations, corresponding to 263 and 308 seconds (4.0 seconds per iteration) on a Cray X-MP/48 for the given initial guesses (0.2 darcies, 1.5, 1.5) and (0.4 darcies, 3.0, 3.0), respectively.

5. Conclusion

A numerical algorithm is developed to estimate the spatially varying absolute permeability, k, and the exponents in the relative permeability expressions for two-phase petroleum reservoirs, based on noisy pressure and flow data. The spatially varying absolute permeability is estimated by regularization with bicubic spline approximation. The algorithm developed suggests the choice of the regularization parameter based on the ratio of the level of the observation error in pressure data to the measure of non-smoothness of parameter. The regularized estimation alleviates the ill-conditioning that resulted from the conventional least-squares estimation. We demonstrate conditions under which the absolute and relative permeabilities can be estimated simultaneously.

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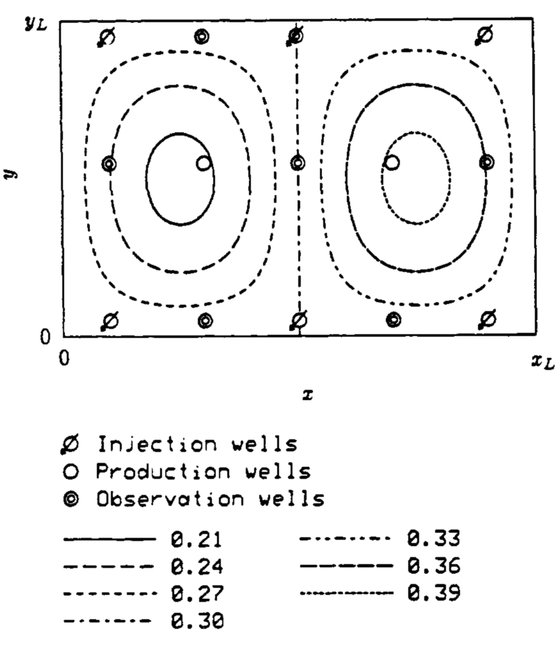

Figure 1 Contours of the assumed true absolute permeability profile and location of wells.

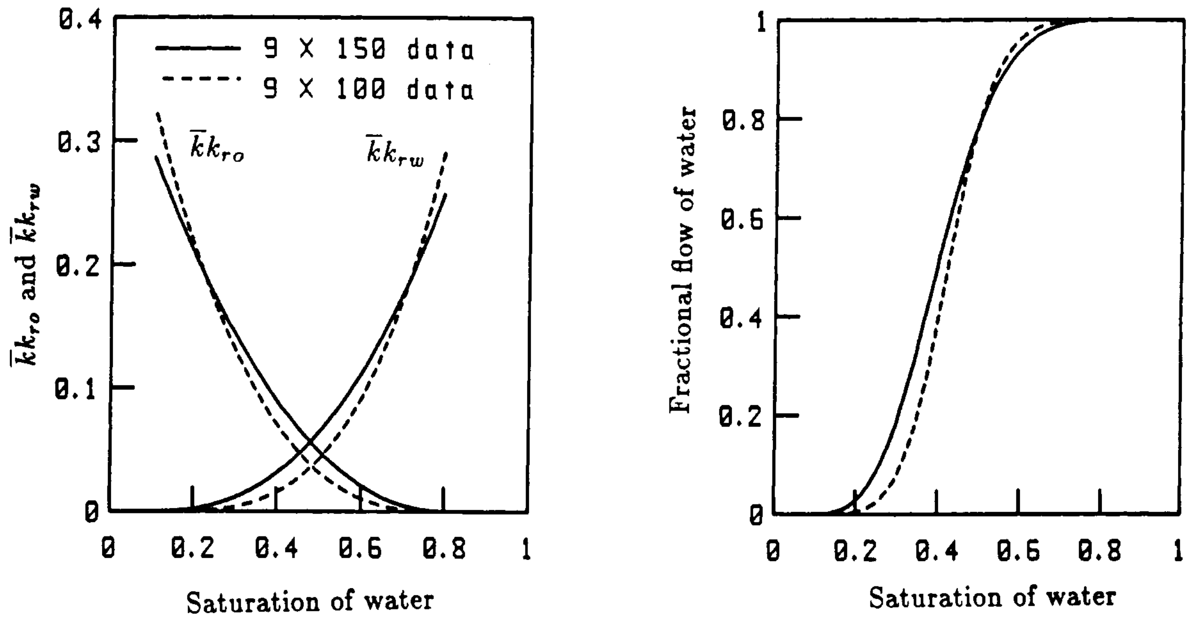

Figure 2 $\bar{k}k_{rw}$, $\bar{k}k_{ro}$, and f_w versus S_w calculated from the resultant $(\bar{k}, b_o, b_w)$'s of Step 1.

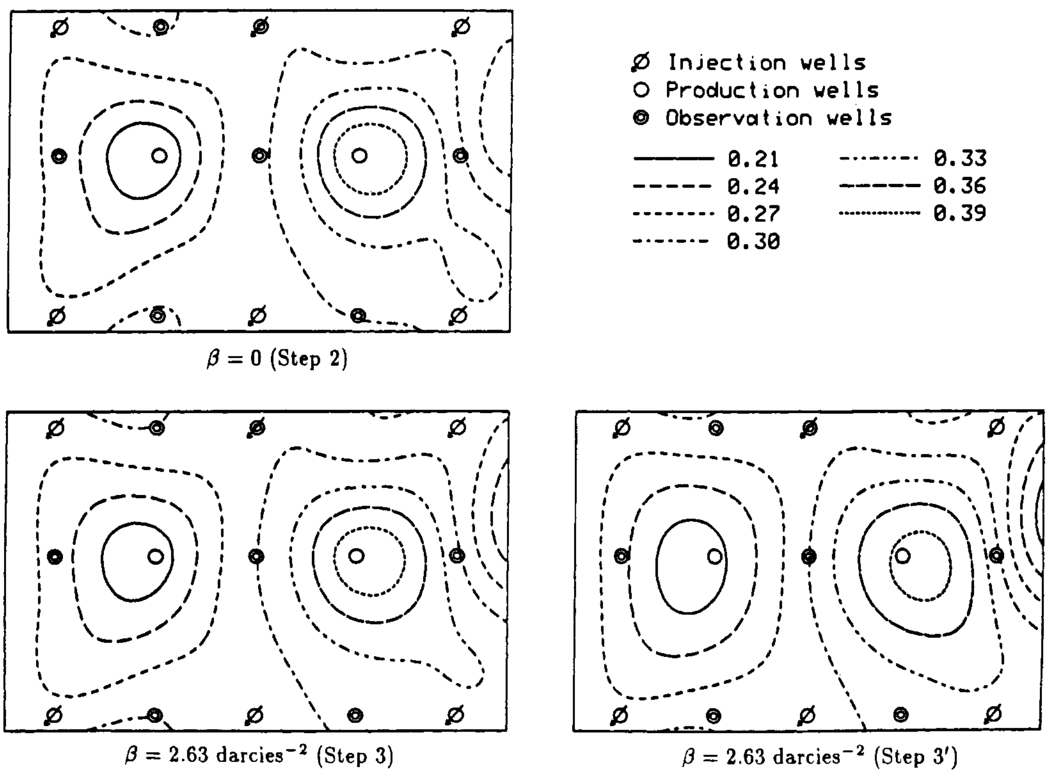

Figure 3 Estimated k surfaces from 150 data points at each well.

Table I Specification of Reservoir Model

Properties of Water and Oil	
$a_w = 0.9$	$a_o = 1.0$
$b_w = 2.5$	$b_o = 2.0$
$S_{iw} = 0.1$	$S_{ro} = 0.2$
$\mu_w = 10^{-3}$ Pa · s	$\mu_o = 3 \times 10^{-3}$ Pa · s
$c_w = 1.94 \times 10^{-9}$ Pa^{-1}	$c_o = 0.97 \times 10^{-9}$ Pa^{-1}
Production Wells	
$q_w = 0.003\ f_w$ m^3/s	$q_o = 0.003\ (1 - f_w)$ m^3/s
Injection Wells	
$q_w = 0.001$ m^3/s	$q_o = 0$

Properties of Reservoir	
c_f	$= 2.91 \times 10^{-9}$ Pa^{-1}
ϕ	$= 0.2 - 0.05 \sin(2\pi x/x_L) \sin(\pi y/y_L)$
$x_L \times y_L \times h$	$= 1500 \times 1000 \times 10$ m^3
$p(x, y, 0)$	$= 1.52 \times 10^7$ Pa
$S_w(x, y, 0)$	$= 0.1$

Table II Performance of estimation of (k, b_o, b_w) from 9×150 data

	$\bar{k}$ darcies	b_o	b_w	β darcies^{-2}	σ_p atm	σ_f	J_{LS}	J_{ST} darcies^2	J_{SM}	CPU time[a] s	Number of[b] Iterations
Initial Guess (a)	0.2	1.5	1.5		2.98	0.0896	181				
Step 1 (from a)	0.289	2.09	2.51		2.20	0.0378	59.3			71	19
Initial Guess (b)	0.4	3.0	3.0		2.51	0.0621	104				
Step 1 (from b)	0.286	2.06	2.48		2.20	0.0380	59.6			116	30
Step 2		1.98	2.50	0.0	0.37	0.0095	2.34	0.430	2.34	127	31
Step 3		1.98	2.50	2.63	0.36	0.0089	2.14	0.274	2.87	54	13
Step 3′		1.98	2.50	2.63	0.36	0.0086	2.04	0.239	2.67	65	16
True values		2.0	2.5		0.34	0.0087	2	0.144			

(*a*) On Cray X-MP/48
(*b*) Number of solving state and adjoint PDEs